AGRODEP Technical Note 14

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Spatial Equilibrium Model for AGRODEP

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1. Introduction

Since the declaration of Malabo in 2014, a trade liberalization reform has taken place in Africa, aimed at increasing trade of agricultural goods within the continent. This program is part of a broader engagement by African countries to tackle the combined issues of famine and poverty. Therefore, it is important for African economists to have access to efficient and transparent economic analysis tools, allowing them to evaluate the potential effects of a change in trade costs has on consumers, producers, and on governments (via public revenue). This change in trade costs could, for example, arise from a trade policy reform, from a simplification in administrative or regulatory practices, as well as from a modification in transportation costs.

This technical note aims to introduce a spatial equilibrium model (SEM) to AGRODEP members. Specifically, by using simplified data, a pedagogic tool is developed to explain the design of a spatial equilibrium model. The potential uses of the model, regarding trade policy analysis as well as transportation costs variation analysis, are exemplified by means of two distinct scenarios. Moreover, this technical note provides a specific focus on the process of calibrating the initial data of the model: supply and demand, prices, trade, and transportation costs.

The spatial equilibrium model is a multi-region partial equilibrium model which links producers and consumers from different locations. It allows economists to examine the global economic and trade consequences of diverse trade policies by determining their effects on market, trade and welfare variables. As an example, it evaluates the impact of such policies on supply and demand, producer and consumer prices, volume and direction of trade, consumer and producer surpluses, as well as on world welfare.

A spatial equilibrium model includes different trade costs: transportation costs that can vary due to a change in infrastructure, as well as specific and/or ad valorem tariffs. The model proposed in this technical note is simple and serves as a baseline model from which we could implement more complex representation of trade costs, such as quotas and minimum prices.

A spatial equilibrium model offers different advantages. First, it evaluates trade diversion and trade creation among trading partners. Second, a spatial equilibrium model allows for creation and elimination of trade flows, which cannot be tackled by many models. Lastly, it is useful for understanding potential trade costs and their effects.

However, the use of a spatial equilibrium model requires an accurate representation of the space being studied, as well as disaggregated data on transportation costs. In the version presented in this document, the model can only be employed to study trade relationships for homogenous products. The users shall make changes to this model in order to study trade flows for differentiated products.

In this technical note, we introduce a spatial equilibrium model applied to corn trade among five African countries.[1] Two scenarios are presented. Scenario A reflects the removal of tariffs between regions, which could result from the implementation of a new trade policy aiming at liberalizing trade. In Scenario B, transportation costs on exports of one country to all its trading partners are increased. We can therefore study the effects of a negative shock on trade flows between countries and on the economy of the region.

Use of this model requires collecting data on trade, transportation costs, prices, supply and demand, as well as on behavioral parameters (specifically, supply and demand elasticities). This data comes from different sources, and can include measurement errors or imperfectly reflect their underlying economic variables. For example, it is difficult to obtain transportation costs for every trade flow of interest. Furthermore, some trade flows between countries may be informal, or not properly recorded at customs. All this data must therefore be calibrated in order to perfectly fit the initial model (meaning the pre-shock model).

Different methods exist to calibrate the data. Bouët et al. (2013) use a cross-entropy technique. The model introduced in this document employs a bi-level programming problem (BLPP). This method, first defined by Jansson and Heckelei (2009), was later exploited in the empirical work of Mosnier (2014), and is described in more detail in Section 2.2 of this document.

Finally, it is crucial to note that the data used in the model presented in this paper is only for pedagogic purposes. Hence, the results obtained from this model should not be exploited for any political interpretation.

The technical note is organized as follows. The next section provides an illustration of the model configuration. Section 3 presents the model structure in GAMS. Section 4 introduces the data, and section 5 dispenses information on running the model. Section 6 demonstrates the different scenarios that can be run with the model, while section 7 indicates processes for applying the model to new datasets. Finally, section 8 discusses the limitations of the spatial equilibrium model in general.

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[1] This model is written using the mathematical solver GAMS.

2. Model structure

2.1 *Model*

We consider five interconnected competitive regional markets (Kenya, Tanzania, Uganda, Zambia and Zimbabwe) trading a single homogeneous good with each other. Trade flows are bidirectional, meaning that a country can both import and export the same product.

Each economy is decentralized: economic decisions (supply, consumption, exportation, importation) are made independently by economic agents, acting in their own interests. Producers in each country attempt to sell their goods at the highest possible price, whereas consumers attempt to buy goods at low prices. Thus, we derive all optimality conditions on demand, supply, producer price, consumer price, and trade flows. Those conditions are described in details in Section 2.5.1.

We also assume the functions of supply and demand are linear in each country.

2.2 *Calibration*

We calibrate the net trade and transportation costs matrices, as well as data on consumer and producer prices, using bi-level programming problems (BLPP – see Jansson and Heckelei, 2009). The goal is to determine the parameters of the optimization model described in Section 2.1.

The calibration is implemented in two steps. The first step, corresponding to the inner problem is the minimization of the sum of the trade costs under the market equilibrium constraint. The second step, or outer problem, is the minimization of the sum of the squared deviations of the transportation costs and prices to their observed values (Mosnier, 2014). Finally, the calibration is based on three assumptions:

- Simple stochastic model for prices and trade costs, with additive measurement errors ($p^{obs} = p^{true} + \varepsilon_p$ and $c^{obs} = c^{true} + \varepsilon_c$).

- Errors on regional prices are independent and identically distributed with a variance of $\frac{1}{w_p}$.

- Errors on trade costs are independent and identically distributed across all pairs with a variance of $\frac{1}{w_c}$ (Jansson and Heckelei, 2009).

Figure 1 Model structure
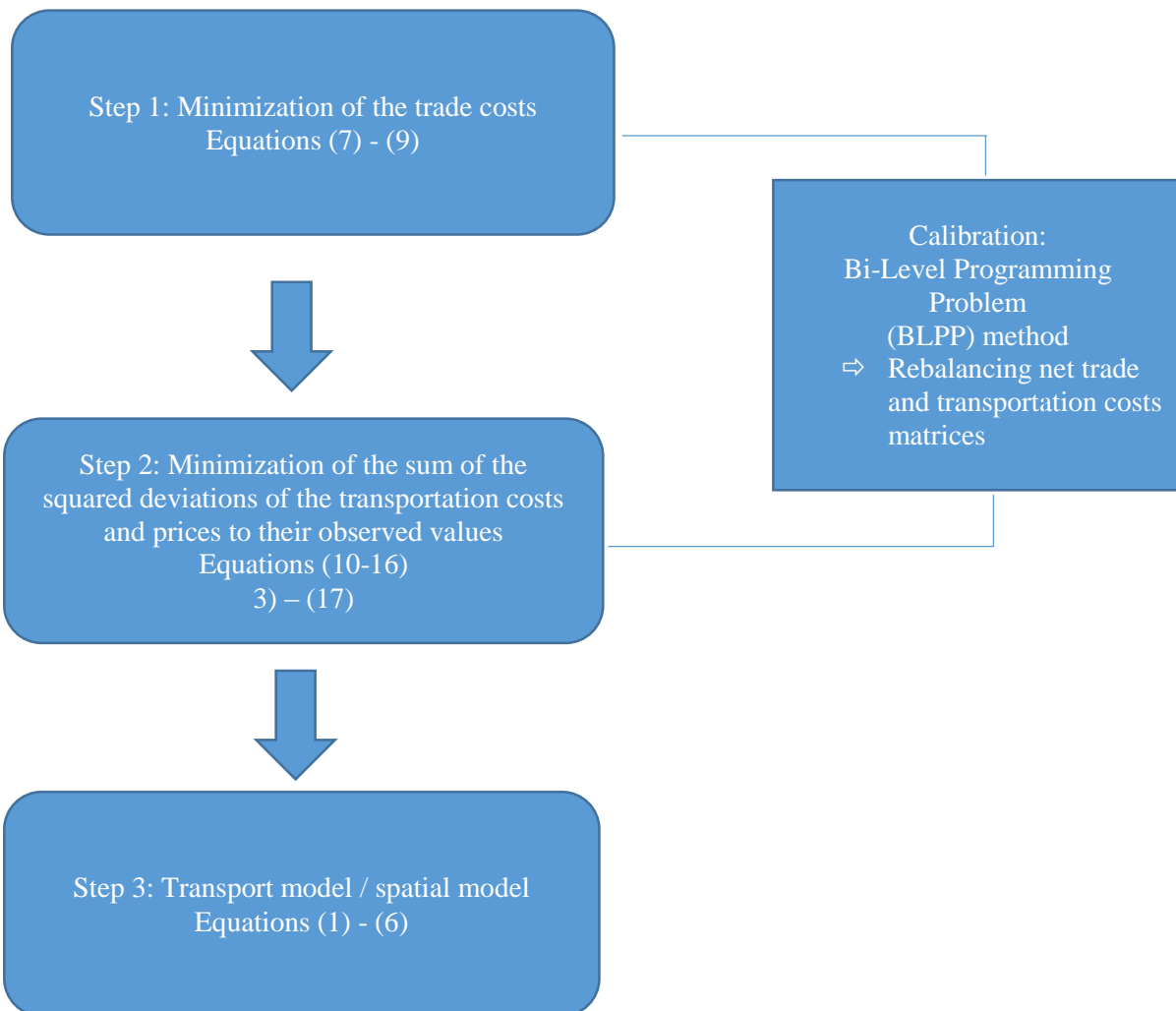
2.3 *Sets*

Regions or countries

- i, exporting country
- j, importing country

2.4 *Parameters and variables*

Table 1 lists the parameters used in this SEM whereas Table 2 presents the variables of the model.

Table 1 Parameters

Parameters	Description
α_i	Positive demand coefficient - Intercept
β_i	Positive demand coefficient - Slope
γ_i	Positive supply coefficient - Intercept
δ_i	Positive supply coefficient - Slope
avt_{ij}	Ad valorem tariffs from i to j
sdt_{ij}	Specific duty tariffs from i to j
c_{ij}	Transportation costs from i to j
w_c	Weight associated with the transport costs
w_p	Weight associated with the prices
μ	Parameter for the complementary slackness condition
e_i	Excess demand in i

Table 2 Variables

Variables	Description
d_i	Quantity demanded in country i
s_i	Quantity produced in country i
x_{ij}	Bilateral trade flows from i to j
xo_{ij}	Observed bilateral trade flows from i to j
ttc	Total trade costs
c_{ij}	Transportation costs from i to j
co_{ij}	Observed transportation costs from i to j
p_i^D	Market demand price in i
p_i^S	Market supply price in i
po_i^S	Observed market supply price in i
p_j^D	Market demand price in j
p_j^S	Market supply price in j
pd_i	Country demand price in i
ps_i	Country supply price in i
π_{ij}	Price chain residual
pen	Penalty
z	Objective function (without the penalty)
zz	Objective function (with the penalty)

2.5 *Equations*

2.5.1 Model

All equations containing tariff terms will be written in two forms: one using ad valorem tariffs (avt_{ij}), and one using specific duty tariffs (sdt_{ij}), in order to add some flexibility regarding tariff data for users. The equation with the ad valorem tariff term (avt_{ij}) will always be presented before the equation with the specific duty tariff term (sdt_{ij}).

Optimality conditions:

The total quantity shipped should be lower than, or equal to, the total quantity produced in each region.

$$\sum_{j=1}^{N} x_{ij} \leq s_i \qquad\qquad \forall i \in [1, N] \qquad\qquad (1)$$

If this inequality strictly holds, market supply price in i is zero ($p_i^S = 0$) whereas it is strictly positive if (1) is an equality.

The total quantity shipped should be greater than, or equal to, the total quantity demanded in each region.

$$\sum_{i=1}^{N} x_{ij} \geq d_j \qquad\qquad \forall j \in [1, N] \qquad\qquad (2)$$

If this inequality strictly holds, market demand price in i is zero ($p_i^D = 0$) whereas it is strictly positive if (2) is an equality.

The country demand price, pd_i, defined by the inverse demand equation ($pd_i = \alpha_i - \beta_i * d_i$), represents the price at which a product can be bought on the market. It should be lower than, or equal to, the market demand price, p_i^D, which represents the price at which consumers are willing to buy a quantity d_i.

$$\alpha_i - \beta_i * d_i \leq p_i^D \qquad\qquad \forall i \in [1, N] \qquad\qquad (3)$$

If this inequality strictly holds, demand in i is zero ($d_i = 0$) whereas it is strictly positive if (3) is an equality.

The country supply price, ps_i, defined by the inverse supply equation ($ps_i = \gamma_i + \delta_i * s_i$), represents the price at which a product can be sold on the market. It should be greater than, or equal to, the market supply price, p_i^S, which represents the price at which producers are willing to sell a quantity s_i.

$$\gamma_i + \delta_i * s_i \geq p_i^S \qquad\qquad \forall i \in [1, N] \qquad\qquad (4)$$

If this inequality strictly holds, supply in i is zero ($s_i = 0$) whereas it is strictly positive if (4) is an equality.

The market supply price in the exporting country i, p_i^S, accounting for transport costs, c_{ij}, and the tariffs, avt_{ij} or sdt_{ij}, should be greater than, or equal to, the market demand price in the importing country j.

If the market supply price in i, accounting for the transportation costs and tariffs, equals the market demand price in j, then there is trade between regions i and j ($x_{ij} > 0$).

If the market supply price in i, accounting for the transportation costs and tariffs, is greater than the market demand price in j, then trade between regions i and j is zero ($x_{ij} = 0$).

$$(1+avt_{ij})(p_i^S+c_{ij}) \geq p_j^D \qquad \forall(i,j) \in [1,N]^2 \qquad (5)$$

or

$$p_i^S+c_{ij}+sdt_{ij} \geq p_j^D$$

The quantity produced, demanded, prices, and bilateral trade flows should be positive or zero.

$$\{s_i \geq 0,\ d_i \geq 0,\ x_{ij} \geq 0,\ p_i^S \geq 0,\ p_j^D \geq 0\} \qquad \forall(i,j) \in [1,N]^2 \qquad (6)$$

2.5.2 Calibration - Step 1

<u>Objective function</u> of the inner problem: minimization of the sum of the total trade costs (ttc).

$$ttc = \Sigma_{ij}(co_{ij}+ po_i^S * avt_{ij})xo_{ij} \qquad (7)$$

or

$$ttc = \Sigma_{ij}(co_{ij}+sdt_{ij})xo_{ij}$$

<u>Constraints:</u>

Market equilibrium constraint:

$$e_i+\Sigma_j(x_{ij}-x_{ji}) = 0 \qquad (8)$$

Bilateral trade flows should be positive or zero.

$$x_{ij} \geq 0 \qquad (9)$$

2.5.3 Calibration - Step 2

<u>Initial objective function</u>: does not include the penalty – minimization of the sum of the squared deviation of the transportation costs and of market supply price to their observed values.

$$z = w_c \Sigma_{ij}(c_{ij} - co_{ij})^2 + w_p \Sigma_i(p_i^S - po_i^S)^2 \qquad (10)$$

<u>Constraints:</u>

Price chain constraint:

$$c_{ij} + p_i^S * (1 + avt_{ij}) = p_j^D \qquad (11)$$

or

$$c_{ij} + sdt_{ij} + p_i^S = p_j^D$$

In order to solve the bi-level programming problem, we use an algorithm based on smooth approximations (Ferris et al. 2002), which allows us to work with larger datasets (Jansson and Heckelei 2004-2009). This algorithm obtained on average the smallest sum of squared errors.

The first step of the algorithm consists in replacing equation (11) by equation (12) which adds a price chain residual, π_{ij}, to the price chain constraint.

$$c_{ij} + p_i^S * (1 + avt_{ij}) = p_j^D + \pi_{ij} \tag{12}$$

or

$$c_{ij} + sdt_{ij} + p_i^S = p_j^D + \pi_{ij}$$

In other words if $\pi_{ij} = 0$, $c_{ij} + sdt_{ij} + p_i^S = p_j^D$ and we obtain $x_{ij} > 0$. If $\pi_{ij} > 0$, $c_{ij} + sdt_{ij} + p_i^S > p_j^D$ and we obtain $x_{ij} = 0$.

Then, the estimation criterion, equation (10), is augmented with a penalty function, equation (13), which leads to the new objective function, equation (14).

$$pen = \mu * \sum_{ij}(\pi_{ij} * x_{ij}) \tag{13}$$

New objective function: including the penalty.

$$zz = z + pen \tag{14}$$

The price chain residual cannot be negative.

$$\pi_{ij} \geq 0 \tag{15}$$

Last, we assume there is no transportation cost on local sales.

$$c_{ii} = 0 \tag{16}$$

3. Model structure in GAMS

We use a systematic code in the declaration of the variables for each step of the program in order to ease the comprehension of the model in GAMS.

The name of the variable is always followed by one or two digits. These digits can only take the value of 1, 2 or 3, and are respectively associated with the first step of the calibration (Calibration 1), the second step of the calibration (Calibration 2), and the model itself (Scenarios A and B). A zero preceding these digits indicates an initial value of the variable.

Example using the variable "trade":

trade01(i,j): indicates the initial value of trade flows from i to j for the first step of the calibration (Calibration 1).

trade1(i,j): indicates the value of trade flows from i to j for the first step of the calibration (Calibration 1).

trade02(i,j): indicates the initial value of trade flows from i to j for the second step of the calibration (Calibration 2).

trade3(i,j): indicates the value of trade flows from i to j after the model (Scenario A or Scenario B) was run.

This reasoning is followed throughout the program in order to facilitate the comprehension of the notations of variables.

3.1 *Model (GAMS)*

Table 3, Table 4, and Table 5 respectively present the model equations under GAMS, the model variables, and the model parameters.

Table 3 Model equations in GAMS

Equations	Definitions	GAMS names
(1)	Quantity produced greater than quantity shipped to all regions	Eq_PROD(i)
(2)	Quantity demanded less than quantity shipped into region	Eq_DEM(i)
(3)	Difference between the market demand price and local demand price	Eq_DPRICEDIF(i)
(4)	Difference between the market supply price and local supply price	EQ_SPRICEDIF(i)
(5)	Price chain constraint 2	PRLINK2

Table 4 Model variables in GAMS

Descriptions	Definitions	GAMS names	Positive variables
d_i	Quantity demanded in country i	demand(i)	
s_i	Quantity produced in country i	supply(i)	
p_i^D	Market demand price in i	consprice3(i)	
p_i^S	Market supply price in i	prodprice3(i)	
x_{ij}	Bilateral trade flows from i to j	trade3(i,j)	

Table 5 Model parameters in GAMS

Definitions	GAMS names
Initial supply in i	supply03(i)
Demand function intercepts p=f(q)	A(i)
Absolute value of demand function slopes p=f(q)	B(i)
Supply function intercepts p=f(q)	C(i)
Absolute value of supply function slopes p=f(q)	D(i)
Additional transport costs from i to j	sup_cost(i,j)
Demand price elasticity of maize	Ed(i)
Supply price elasticity of maize	Es(i)
Parameter used to calibrate B and D in cases where initial supply or initial demand is zero	countd
Parameter used to calibrate B and D in cases where initial supply or initial demand is zero	counts
Parameter used so that the complementarity relationships are verified	epsilon

3.2 *Calibration - Step 1 (GAMS)*

Table 6, Table 7 and Table 8 respectively present the equations, variables, and parameters used in the first step of the calibration under GAMS.

Table 6 Equations – Step 1 in GAMS

Equations	Definitions	GAMS names
(7)	Sum of the trade costs	Eq_TTC
(8)	Market equilibrium constraint	Eq_Mkt(i)

Table 7 Variables – Step 1 in GAMS

Descriptions	Definitions	GAMS names	Positive variables
xo_{ij}	Matrix of the net trade from i to j (in KG)	trade1(i,j)	
ttc	Total trade costs from i to j (rebalanced)	TTC	

Table 8 Parameters – Step 1 in GAMS

	Definitions	**GAMS names**
Net trade parameters		
	Initial matrix of net trade from i to j (in KG)	trade01(i,j)
Transport costs parameters		
	Initial matrix of transport costs from i to j (in $ per ton)	transportcost1(i,j)
	Ad valorem tariff between regions i and j	avt(i,j)
	Specific duty tariff between regions i and j	sdt(i,j)
Additional parameters		
	Initial producer price	prodprice1(i)
	Initial total trade costs from i to j	TTC0
	Excess demand in i	e(i)

3.3 *Calibration - Step 2 (GAMS)*

Table 9, Table 10 and 1Table 11 respectively present the equations, variables, and parameters used in the second step of the calibration under GAMS.

Table 9 Equations – Step 2 in GAMS

Equations	**Definitions**	**GAMS names**
(10)	Objective function (without the penalty)	Eq_Obj
(12)	Price chain constraint 1	Eq_PRLINK1(i,j)
(13)	Penalty equation	Eq_Pen
(14)	New objective function (including the penalty)	Eq_zz
(16)	Transport costs on local sales are zero	Eq_tc

Table 10 Variables – Step 2 in GAMS

Descriptions	**Definitions**	**GAMS names**	**Positive variables**
c_{ij}	Transportation costs	transportcost2(i,j)	
p_i^S	Market supply price	prodprice2(i)	
p_i^D	Market demand price	consprice2(i)	
π_{ij}	Price chain residual	pi(i,j)	
pen	Penalty	pen	
z	Objective function (without the penalty)	z	
zz	New objective function (including the penalty)	zz	

Table 11 Parameters – Step 2 in GAMS

	Definitions	GAMS names
Parameters related to the initial objective function		
	Weight associated with the transportation costs	wtc
	Weight associated with the market supply price	wp
Parameters related to the price linkage equation		
	Initial demand	demand02(i)
	Initial consumer price	consprice02(i)
Parameters related to the penalty function		
	Parameter for the complementary slackness condition	mu
	Initial price chain residual	pi0(i,j)
	Initial penalty	pen0
Parameters related to the new objective function		
	Initial objective function (without the penalty)	z0
	Initial new objective function (including the penalty)	zz0

4. Data structure

For pedagogic purposes, we have limited the data to a set of five countries and to one commodity (maize). Table 12 lists these five countries as well as their ISO Alpha-3 code.

Table 12 Countries in the model

ISO	Country
KEN	Kenya
TZA	Tanzania
UGA	Uganda
ZMB	Zambia
ZWE	Zimbabwe

Data matrices employed in this model are presented in Table 13, Table 14 and Table 15. They were all obtained from Bouët et al. (2013) but were simplified to reflect the simple geographic setting of only five countries.

Table 13 Net trade matrix (in tons)

	KEN	TZA	UGA	ZMB	ZWE
KEN	15200000	0	0	339711	252850
TZA	4008548	2555000	0	1823739	165334
UGA	3472272	4229010	1350000	2882916	295967
ZMB	0	0	0	1250000	10171301
ZWE					

Source: Bouët et al. (2013) and author's calculations

Table 14 Transportation costs matrix (USD/ton)

	KEN	TZA	UGA	ZMB	ZWE
KEN	0	4.428969	3.92537	11.68115	13.91303
TZA	4.428969	0	6.389507	9.045604	10.7874
UGA	3.92537	6.389507	0	11.5714	14.56105
ZMB	11.68115	9.045604	11.5714	0	3.925581
ZWE	13.91303	10.7874	14.56105	3.925581	0

Source: Bouët et al. (2013) and author's calculations

Table 15 Ad valorem tariffs matrix

	KEN	TZA	UGA	ZMB	ZWE
KEN	0	0.125	0.014	0	0
TZA	0.25	0	0	0.05	0.25
UGA	0.05	0	0	0.01	0.05
ZMB	0	0.125	0.014	0	0
ZWE	0	0.125	0.014	0	0

Source: Bouët et al. (2013) and author's calculations

In Table 15, we present tariffs in their ad valorem form. The model can be run using specific duty tariffs (see the beginning of Calibration 1.gms program, page 35). The specific duty tariffs can be calculated from the ad valorem tariffs by using the following equation, and vice versa:

$$po_i^S + sdt_{ij} = po_i^S * (1 + avt_{ij})$$

Table 16 Specific duty tariffs matrix

	KEN	TZA	UGA	ZMB	ZWE
KEN	0	22.85284	2.559518	0	0
TZA	45.70568	0	0	9.141135	45.70568
UGA	9.141135	0	0	1.828227	9.141135
ZMB	0	22.85284	2.559518	0	0
ZWE	0	24.50329	2.744368	0	0

Source: Bouët et al. (2013) and author's calculations

The data used came from the study by Bouët et al. (2013). The table below provides more precise information regarding the various sources of the data.

Table 17 Data sources

Parameters	Years	Sources of original data
Production	1995-2005	FAOSTAT, UN Food and Agricultural Organization.
Domestic prices	1995-2005	FAOSTAT, UN Food and Agricultural Organization.
Consumer prices	1995-2005	FAOSTAT, UN Food and Agricultural Organization.
Elasticities of supply	2001-2005	IMPACT model, International Food Policy Research Institute.
Elasticities of demand	2001-2005	IMPACT model, International Food Policy Research Institute.
Net trade flows	1995-2005	UN COMTRADE 1005 (HS-4) bilateral trade data.
Transportation costs	2004-2006	Ocean freight rates from the International Grains Council.
Ad valorem tariffs	2005	MAcMap HS-6.

5. Running the model

Figure 2 GAMS files
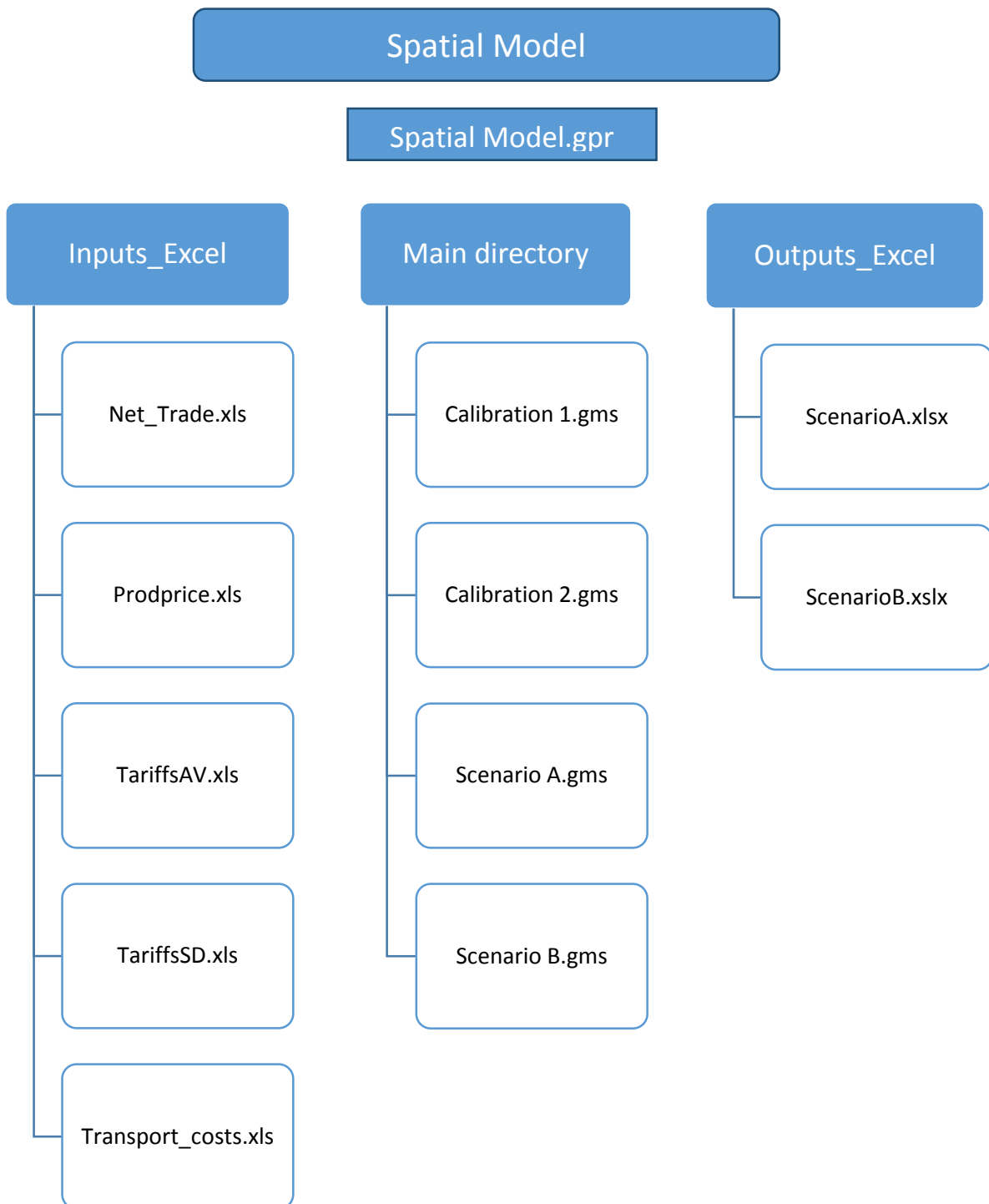
Data is located in five Excel files, situated in the same file called « Inputs_Excel ».

- Net_Trade.xls (includes bilateral trade flows between regions)
- Prodprice.xls (includes producer prices for each region)
- TariffsAV.xls (includes ad valorem tariffs between regions)
- TariffsSD.xls (includes specific duty tariffs between regions)
- Transport_costs.xls (includes transportation costs between regions)

Four GAMS programs are included in the main directory and allow us to run the spatial model. Two of them are used in the calibration of the data. The other two present the scenarios developed in this technical note.

- Calibration 1 (calls the program to run the first step of the BLPP calibration)
- Calibration 2 (calls the program to run the second step of the BLPP calibration, using the results from the first one)
- Scenario A (calls the program to run the first scenario of the model, presented on page 22)
- Scenario B (calls the program to run the second scenario of the model, presented on page 30)

Finally, the results are presented in two Excel files, one for each scenario.

- ScenarioA.xlsx (gathers the results from Scenario A on market variables, trade variables and welfare variables, on separated sheets)
- ScenarioB.xlsx (gathers the results from Scenario B on market variables, trade variables and welfare variables, on separated sheets)

In order to run the model using this data, users must:

1. Create a new project file "Spatial Model.gpr" in GAMS-IDE. This project file should be located in the same directory as all other files.
2. Open the program "Calibration 1.gms". Press on the button with the red arrow to run the program. In "Calibration 1.gms", files are created that are then used as inputs to other files. Users can also recall an already saved file. The necessary save and restart commands for each file run are already included in "Calibration 1.gms". "Calibration 1.gms" output is saved with the command, "s=Calibration 1" (Figure 3 "Calibration 1.gms" file).
3. Open the program "Calibration 2.gms". Press on the button with the red arrow to run the program. The "Calibration 1.gms" output is retrieved by "Calibration 2.gms", with the command "r=Calibration 1". The "Calibration 2.gms" output is also saved using the command "s=Calibration 2" (Figure 4).

Figure 3 "Calibration 1.gms" file
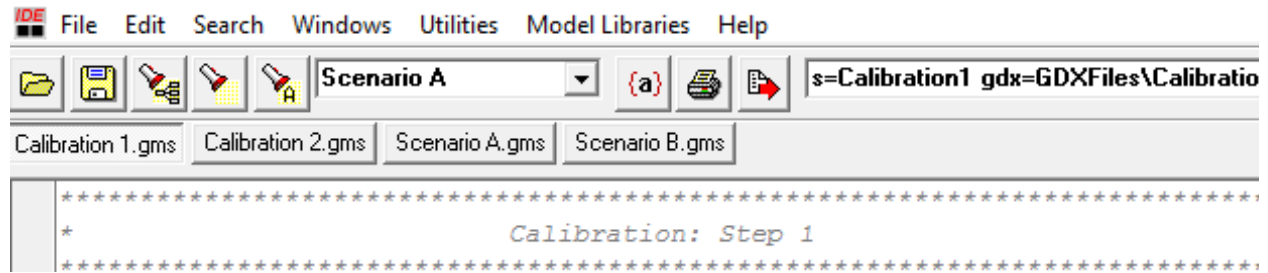
Figure 4 "Calibration 2.gms" file
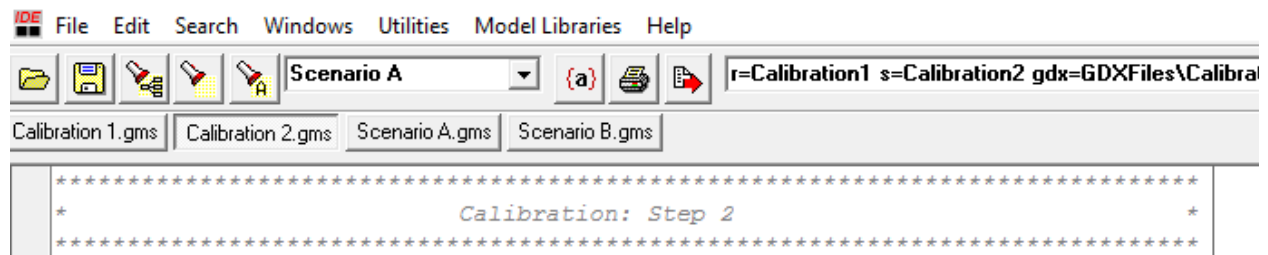
The gdx files created in "Calibration 1.gms" and "Calibration 2.gms", respectively called "Calibration1.gdx" and "Calibration2.gdx", allow us to have a global perspective of our results, more particularly, of each parameters and variables for each step of the program. They are automatically generated with the command "gdx=GDXFiles\Name", where "GDXFiles" allows to store all gdx files in the same folder, called GDXFiles.

6. Results

We study two scenario. The first one representing trade liberalization, and the second one representing an increase in transportation costs from one country.

6.1 *Scenario A*

Scenario A consists in an annulation of all tariffs.

Run Scenario A in GAMS:

1. Open the program "Scenario A.gms". Run the program. Similarly, the "Calibration 2.gms" output is retrieved by "Scenario A.gms" with the command "r=Calibration2". The "Scenario A.gms" output is saved with the command "s=ScenarioA" (Figure 5).

Figure 5 "Scenario A.gms" file

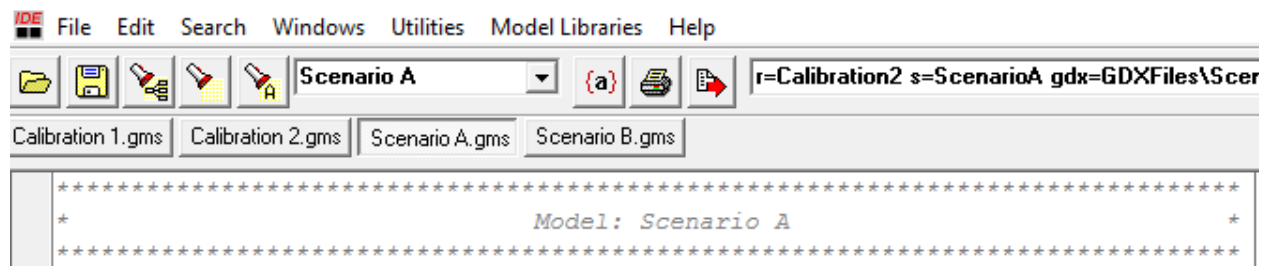2. The command "gdx=GDXFiles\ScenarioA" creates a gdx file that presents the values of all variables and parameters included in the model. From this file, an Excel file was completed, "ScenarioA.xlsx", summarizing the results.

The results obtained from Scenario A are presented below. Initial and final matrices of trade are given by Table 18 Net trade matrix - Baseline and Table 19.

Table 18 Net trade matrix - Baseline

Net trade matrix - Baseline							
	KEN	**TZA**	**UGA**	**ZMB**	**ZWE**	**Exports**	
KEN	15200000	0	0	0	0	0	
TZA	0	2555000	0	1768611	0	1768611	
UGA	6888259	0	1350000	3991906	0	10880165	
ZMB	0	0	0	1250000	10885452	10885452	
ZWE	0	0	0	0	0	0	
Imports	6888259	0	0	5760517	10885452	23534228	Exports + Imports
						20355000	Sum of local sales
						43889228	Total trade

Source: author's calculations

Table 19 Net trade matrix – Scenario A

Net trade matrix - Scenario A							
	KEN	**TZA**	**UGA**	**ZMB**	**ZWE**	**Exports**	
KEN	11904207	2545956	0	0	0	2545956	
TZA	0	0	0	4670954	0	4670954	
UGA	10278916	0	1349944	906296	0	11185212	
ZMB	0	0	0	1432285	10885345	10885345	
ZWE	0	0	0	0	0	0	
Imports	10278915	2545956	0	5577250	10885345	29287467	Exports + Imports
						14686435	Sum of local sales
						43973902	Total trade

Source: author's calculations

Table 20 indicates the variation in trade flows for each possible line.

Table 20 Net trade variation: Baseline – Scenario A

Variation in net trade: Baseline - Scenario A							
	KEN	**TZA**	**UGA**	**ZMB**	**ZWE**	**Exports**	
KEN	-21.68%	100.00%	0.00%	0.00%	0.00%	100.00%	
TZA	0.00%	-100.00%	0.00%	164.10%	0.00%	164.10%	
UGA	49.22%	0.00%	0.00%	-77.30%	0.00%	2.80%	
ZMB	0.00%	0.00%	0.00%	14.58%	0.00%	0.00%	
ZWE	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	
Imports	49.22%	100.00%	0.00%	-3.18%	0.00%	24.45%	Δ(Exports + Imports)
						-27.85%	Δ(Sum of local sales)
						0.19%	ΔTotal trade

Source: author's calculations

Figure 6 Variations in trade flows: Baseline – Scenario A

Values of trade flows before the shock

Values of trade flows after the shock

(%): variation in trade flows from the shock

KENYA
ΔSupply: -4.93%
ΔDemand: +0.43%

ZIMBABWE
ΔSupply: 0%
ΔDemand: 0%

6888259
10278916
(+49.22%)

0
11904206
(+100%)

10885452
10885345
(0%)
%)

TANZANIE
ΔSupply: +8.03%
ΔDemand: -0.35%

1768611
4670954
(+164%)

ZAMBIE
ΔSupply: +1.5%
ΔDemand: -0.01%

3991906
906296
(-77.3%)

UGANDA
ΔSupply: +2.49%
ΔDemand: 0%
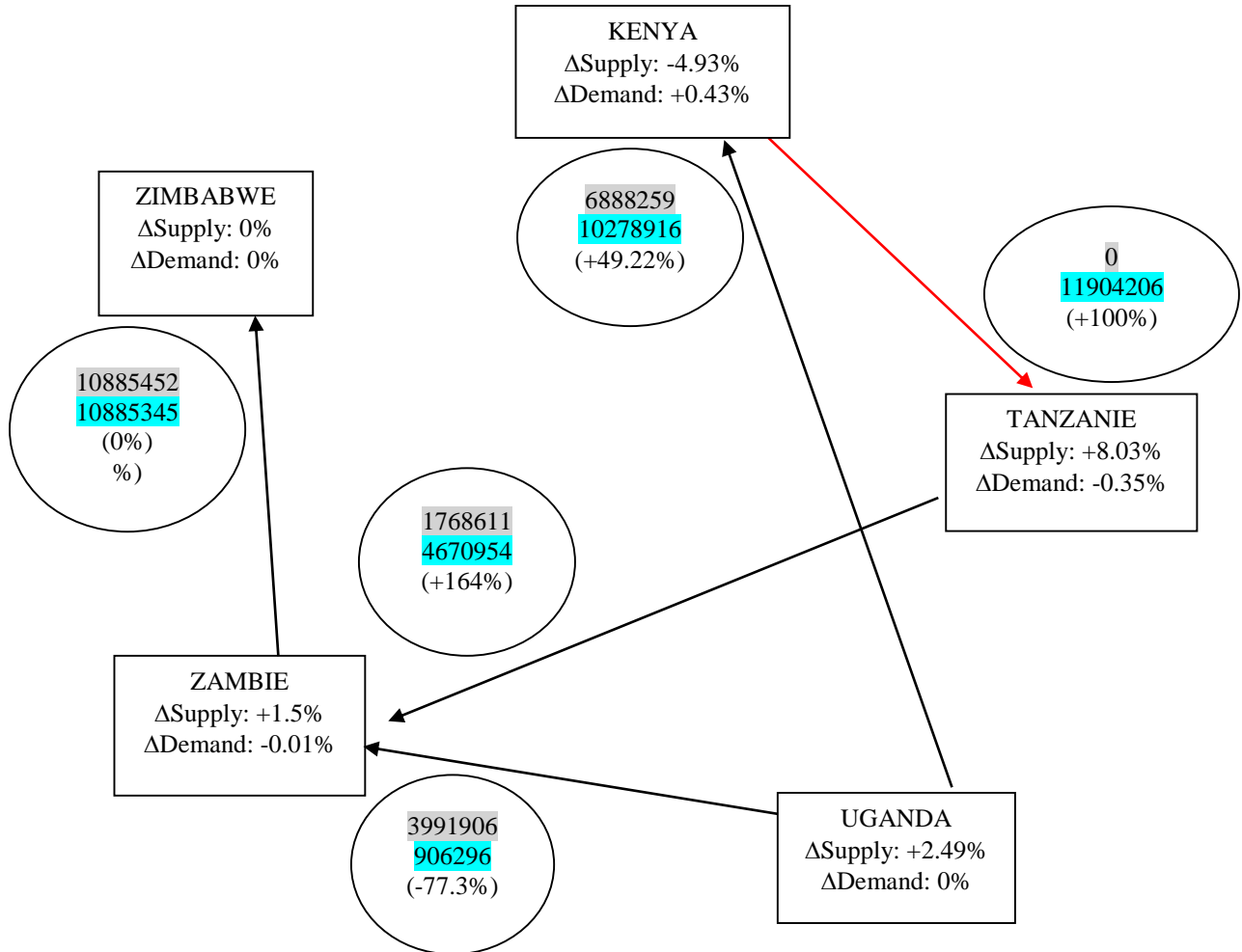
Source: author's calculations

Table 21 and Table *22* present the values of market variables (supply, consumption and prices) respectively before and after the shock.

Table 21 Market variables - Baseline

Market variables - Baseline				
	Supply	**Consprice**	**Prodprice**	**Demand**
KEN	15200000	187.3722	187.3722	22088259
TZA	4323611	178.2732	178.2732	2555000
UGA	12230165	178.2311	178.2311	1350000
ZMB	12135452	187.4143	187.4143	7010517
ZWE	0	191.3399	196.0263	10885452
Total/Avg	43889228	185	185	43889228

Source: author's calculations

Table 22 Market variables – Scenario A

Market variables - Scenario A				
	Supply	**Consprice**	**Prodprice**	**Demand**
KEN	14450162	181.9349	181.9349	22183122
TZA	4670954	186.3639	189.29	2545955.5
UGA	12535156	181.9349	181.9349	1349943.9
ZMB	12317630	189.29	189.29	7009534.7
ZWE	0	193.2156	196.0263	10885345
Total/Avg	43973902	187	188	43973902

Source: author's calculations

Table 23 presents the variation associated with market variables.

Table 23 Variation in market variables: Baseline – Scenario A

Variation in market variables: Baseline-Scenario A				
	Supply	**Consprice**	**Prodprice**	**Demand**
KEN	-4.93%	-2.90%	-2.90%	0.43%
TZA	8.03%	4.54%	6.18%	-0.35%
UGA	2.49%	2.08%	2.08%	0.00%
ZMB	1.50%	1.00%	1.00%	-0.01%
ZWE	0.00%	0.98%	0.00%	0.00%
Total/Avg	0.19%	1.10%	1.20%	0.19%

Source: author's calculations

From variations in market variables and considering the hypothesis of linear supply and demand functions, we can calculate consumer and producer surpluses, associated with this trade reform. Those surpluses, as

well as public revenue (tariff revenue) are indicated in Table 24 (before the shock) and Table 25 (after the shock).

Table 24 Welfare variables – Baseline

Welfare variables - Baseline				
	CoS	**PrS**	**PR**	**Welfare**
KEN	13982180305	837663890	62966505	14882810700
TZA	2919795270	296455396	0	3216250666
UGA	60152979658	908247983	0	61061227642
ZMB	46923981458	758119279	23465222	47705565960
ZWE	1.04E+12	91716990	0	1.04E+12
Total	1.16539E+12	2892203538	86431727	1.16828E+12

Source: author's calculations

Table 25 Welfare variables – Scenario A

Welfare variables - Scenario A				
	CoS	**PrS**	**PR**	**Welfare**
KEN	14102537802	757056076	0	14859593877
TZA	2899160168	346000984	0	3245161152
UGA	60147979531	954111814	0	61102091345
ZMB	4.69E+10	781051971	0	47691885086
ZWE	1.04E+12	91694058	0	1.04E+12
Total	1.16545E+12	2929914903	0	1.16829E+12

Source: author's calculations

Table 26 indicates variations (in US$) in consumer and producer surpluses as well as variations in public revenue for each country and for the overall region. Finally, variation in world welfare is presented in the last column, representing the sum of the three preceding elements.

Table 26 Variation in welfare variables: Baseline-Scenario A

Variation in welfare variables: Baseline-Scenario A				
	CoS	**PrS**	**PR**	**Welfare**
KEN	120357497	-80607814	-62966505	-0.16%
TZA	-20635102	49545588	0	0.90%
UGA	-5000127	45863831	0	0.07%
ZMB	-13148343	22932692	-23465222	-0.03%
ZWE	-20417180	-22932	0	0.00%
Total	61156745	37711365	-86431727	0.00107%

Source: author's calculations

The annulation of tariffs has a great impact on the allocation of trade flows between countries. Trade creation and trade distortion rise because of the annulation of tariffs (Source: author's calculations

Figure 6). A trade flow is for example created between Kenya and Tanzania. One of the explanations resides in the asymmetric transportation costs between countries. The transport costs on exports from Uganda to Kenya are zero, whereas the ones from Kenya to Uganda are positive. Hence, Kenya may be better off importing maize from Uganda. However, it does not export any of its production of maize to Uganda. Kenya rather exports towards Tanzania because transportation costs between these two countries are zero.

Zambia, whose main trading partner before the shock was Uganda, decreases its imports from Uganda in favor of imports from Tanzania. Imports from Tanzania to Uganda hence increase by 164% (Table 20). We also observe a rise in trade between Uganda and Kenya by 49.22 (Table 20). From a global perspective, trade between countries increases by 0.19% thanks to this new trade policy. The reallocation of trade flows has a different impact on supply and demand, as well as on producer and consumer prices in each country.

As an example, the increase in exports from Kenya is offset by an increase in its imports. As a result, supply in Kenya decreases, followed by a decrease in consumer and producer prices, resulting in a slight increase in local demand (Table 23). Local sales decrease by 21.68% (Table 20). A potential explanation resides in the fact that producer prices in Uganda are equal to consumer prices in Kenya. In addition, we know that transport costs between these countries are zero, and tariffs are zero. Hence, consumers in Kenya are indifferent regarding their decision of buying locally-produced maize or maize imported from Uganda.

In other countries, supply increases from the rise in exports, resulting in a rise in consumer and producer prices. The impacts on local sales vary between countries. In Tanzania, local sales are zero: the entire production is exported towards Zambia (Table 19). Transportation costs between these countries are zero and consumer price in Zambia equals producer price in Tanzania (Table 22). Tanzania imports its maize from Kenya because transportation costs between these countries are lower than those between Tanzania and other countries. Furthermore, consumer price in Tanzania equals the sum of producer price in Kenya and of transportation costs between these countries. Consumer price in Tanzania is also lower than its producer price. Consequently, consumers in Tanzania have more incentives to buy maize from Kenya rather than from local producers. On the contrary, we observe an increase in local sales in Zambia, which could be explained by the decrease in its imports (Table 20).

In terms of welfare, consumer surplus in Kenya increases, whereas its producer surplus lowers. In addition, the removal of tariffs results in the annulation of public revenue in Kenya. Hence, Kenya's welfare decreases by 0.16% (Table 26). In Tanzania, Uganda, and Zambia, consumer surpluses decrease in favor of producer surpluses, resulting in an increase in welfare in Tanzania and Uganda. In Zambia, public revenue decreases, leading to a decrease in welfare. Finally, the loss in consumer surplus in Zimbabwe offsets the gain in producer surplus, leading to a relatively constant welfare (Table 26). Overall welfare is barely impacted (it increases by 0.0010%), demonstrating that the new policy has a limited effect on global welfare in the area under study (Table 26).
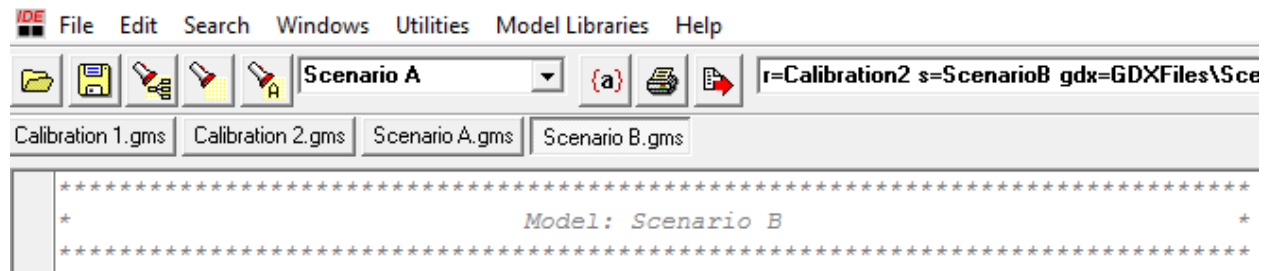
6.2 *Scenario B*

Scenario B consists in an increase in transportation costs on Uganda's exports to all its trading partners, from 0 to 50 USD per ton.

Run Scenario B in GAMS:

1. Open the program "Scenario B.gms". Run the program. Similarly, the "Calibration 2.gms" output is retrieved by "Scenario B.gms" with the command "r=Calibration2". The "Scenario B.gms" output is saved with the command "s=ScenarioB" (Figure 7).

Figure 7 "Scenario B.gms" file

2. The command "gdx=GDXFiles\ScenarioB" creates a gdx file that presents the values of all the variables and parameters included in the model. From this file, an Excel file was completed, "ScenarioB.xlsx", summarizing the results.

The results obtained from Scenario B are presented following the same order as results from Scenario A in Table 27 to Table 32, and in

Figure 8.

Table 27 Net trade matrix – Scenario B

Net trade matrix - Scenario B							
	KEN	**TZA**	**UGA**	**ZMB**	**ZWE**	**Exports**	
KEN	16608109	0	0	0	0	0	
TZA	0	2543586	0	2101949	0	2101949	
UGA	5302008	0	1350603	2301144	0	7603152	
ZMB	0	0	0	2602077	10525105	10525105	
ZWE	0	0	0	0	359766	0	
Imports	5302008	0	0	4403093	10525105	20230206	Exports + Imports
						234641407	Sum of local sales
						43694346	Total trade

Source: author's calculations

Table 28 Net trade variations: Baseline – Scenario B

| Variation in net trade: Baseline - Scenario B							
	KEN	**TZA**	**UGA**	**ZMB**	**ZWE**	**Exports**	
KEN	9.26%	0.00%	0.00%	0.00%	0.00%	0.00%	
TZA	0.00%	-0.45%	0.00%	18.85%	0.00%	18.85%	
UGA	-23.03%	0.00%	0.04%	-42.35%	0.00%	-30.12%	
ZMB	0.00%	0.00%	0.00%	108.17%	-3.31%	-3.31%	
ZWE	0.0000%	0.00%	0.00%	0.00%	0.00%	0.00%	
Imports	-23.03%	0.00%	0.00%	-23.51%	-3.31%	-14.04%	Δ(Exports+Imports)
						15.27%	Δ(Sum of local sales)
						-0.44%	ΔTotal trade

Source: author's calculations

Figure 8 Variations in trade flows: Baseline – Scenario B

Values of trade flows before the shock

Values of trade flows after the shock

(%): variation in trade flows from the shock
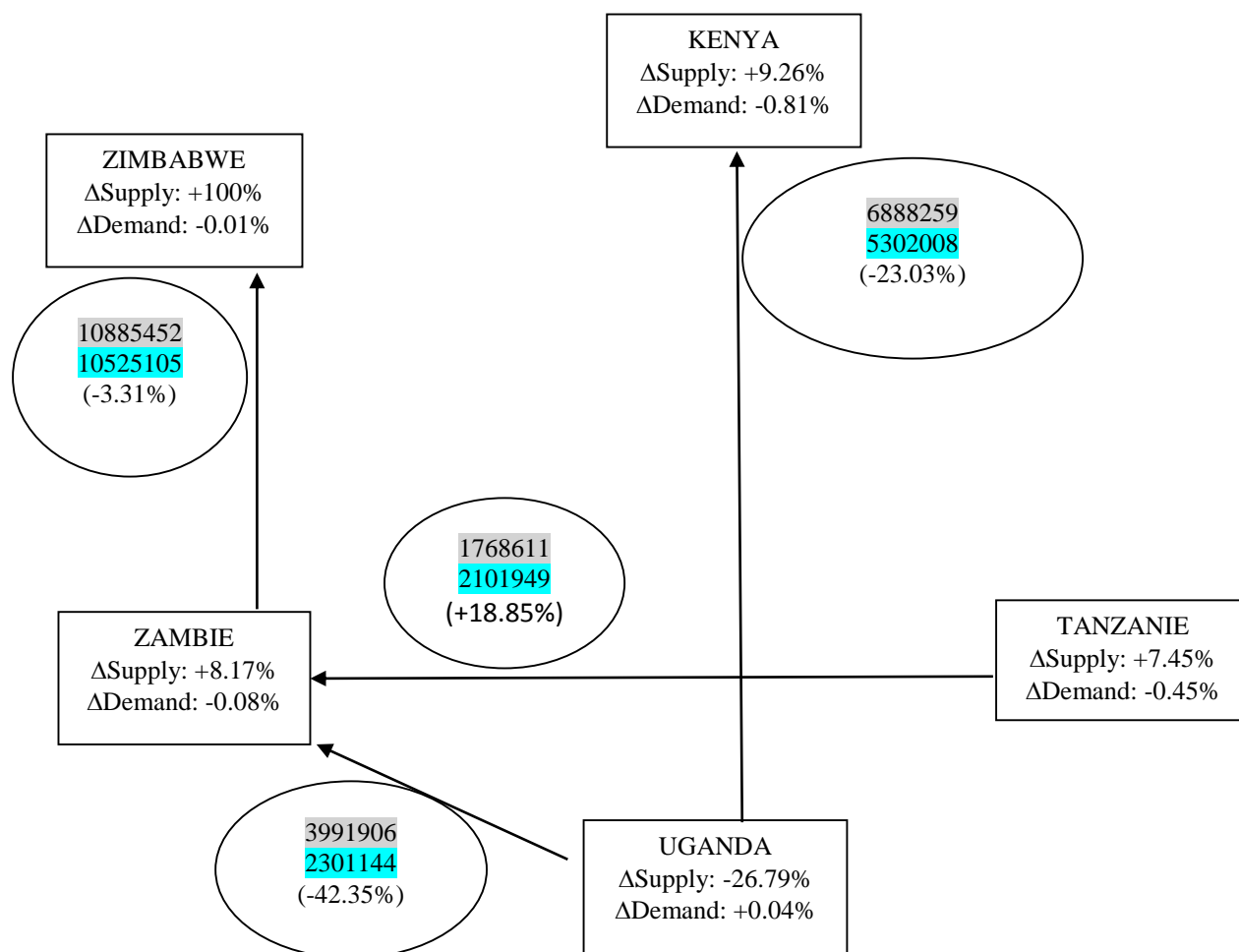
Source: author's calculations

Table 29 Market variables – Scenario B

Market variables - Scenario B				
	Supply	**Consprice**	**Prodprice**	**Demand**
KEN	16608109	197.5827	197.5827	21910117
TZA	4645535	188.4838	188.4838	2543585.7
UGA	8953755	138.4416	138.4416	1350602.8
ZMB	13127181	197.6249	197.6249	7005169.8
ZWE	359766	201.5505	201.5505	10884871
Total/Avg	43694346	185	185	43694346

Source: author's calculations

Table 30 Variations in market variables: Baseline-Scenario B

Variation in market variables: Baseline-Scenario B				
	Supply	**Consprice**	**Prodprice**	**Demand**
KEN	9.26%	5.45%	5.45%	-0.81%
TZA	7.45%	5.73%	5.73%	-0.45%
UGA	-26.79%	-22.32%	-22.32%	0.04%
ZMB	8.17%	5.45%	5.45%	-0.08%
ZWE	100.00%	5.34%	2.82%	-0.01%
Total/Avg	-0.44%	0.11%	-0.39%	-0.44%

Source: author's calculations

Table 31 Welfare variables – Scenario B

Welfare variables - Scenario B				
	CoS	**PrS**	**PR**	**Welfare**
KEN	13757556659	1000052914	48466371	14806075944
TZA	2893765607	342245317	0	3236010924
UGA	60206707418	486799638	0	60693507056
ZMB	46852427574	887091877	23421212	47762940663
ZWE	1041299669080	993526	0	1041300662600
Total	1165010126338	2717183272	71887583	1167799197187

Source: author's calculations

Table 32 Variation in welfare variables: Baseline-Scenario B

Variation in welfare variables: Baseline-Scenario B				
	CoS	**PrS**	**PR**	**Welfare**
KEN	-224623646	162389024	-14500134	-0.52%
TZA	-26029663	45789921	0	0.61%
UGA	53727760	-421448345	0	-0.60%
ZMB	-71553884	128972598	-44010	0.12%
ZWE	-111143400	-90723464	0	-0.01%
Total	-379622833	-175020266	-14544144	-0.04087%

Source: author's calculations

The increase in transportation costs on exports from Uganda to all its partners modifies the structure of trade between countries. As expected, exports from Uganda decrease, by 23.03% towards Kenya and by 42.35% towards Zambia, leading to a total decrease of Uganda's exports by 30.12% (Table 28 Net trade variations: Baseline – Scenario Band

Figure 8). Therefore, supply in Uganda lowers, leading to a decrease in producer and consumer prices, as well as in a slight increase in demand. This last phenomenon leads to a slight increase in local sales (+0.04%) (Table 28). Consequently, producer surplus decreases and is not offset by the increase in consumer surplus, leading to a decrease in welfare in Uganda by 0.6% (Table 32).

The shock also has consequences on other countries. Imports from Kenya and from Zambia decrease respectively by 22.03% and by 23.51% (Table 28). Zambia imports more maize from Tanzania. Transportation costs between Zambia and Tanzania, as well as between Zambia and Zimbabwe, are zero. Hence, Zambia may have incentives to import from those two countries. Yet, the sum of producer price in Tanzania and of tariffs between Zambia and Tanzania equals consumer price in Zambia. However, the sum of producer price in Zimbabwe and of tariffs from Zambia to Zimbabwe are higher than consumer price in Tanzania (Table 29). Consequently, Tanzania chooses to increase its imports from Tanzania, rather than from Zimbabwe.

Supply increases in Kenya (+9.26%), in Tanzania (+7.45%), in Zambia (+8.17%) and more particularly in Zimbabwe (+100%) where supply was zero before the shock (Table 30). The rise in supply results in an increase in prices and in a slight decrease in local demands (Table 30). Hence, local sales in Tanzania slightly decrease. On the contrary, local sales in Kenya and in Zambia largely increase due to their reduced imports (Table 28). Finally, consumer surpluses in each country (except Uganda) decrease, whereas producer surpluses increase (except in Zimbabwe and in Uganda). It is important to note as well the loss in public revenue in Kenya and in Zambia, due to their decrease in imports. Results in terms of welfare vary from one country to another, but stay relatively weak (Table 32).

From a global perspective, trade among countries decreases by 0.44% (Table 28). Global consumer and producer surpluses lower as well. We also observe a decrease in world public revenue (led by those of Kenya and Uganda). These three impacts lead to a decrease in global welfare in the region by 0.0409% (Table 32).

As previously mentioned, the results are displayed for a pedagogic purpose only, and should not be elements of argumentations in favor of any political policy. However, they clearly demonstrate how a spatial equilibrium model can be used in order to understand the potential impact of trade policies on key variables. They also display one of the main advantages of the model: its ability to allow for the creation and elimination of trade flows between countries.

7. Update of model with new data

The spatial equilibrium model developed in the technical note can be exploited by trade policy analysts to study other trade policy scenarios, other countries, or other commodities.

7.1 *Applying the model to different tariff equivalents*
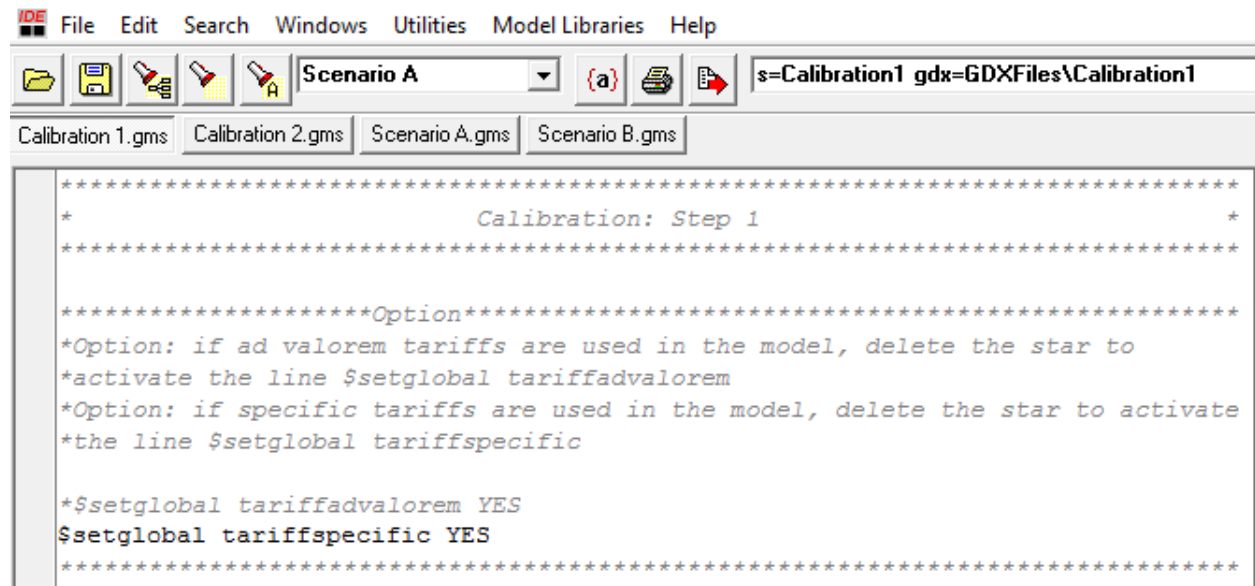
The model presented in this document employs specific duty tariffs. However, depending on the data available, ad valorem tariffs could also be used in a very simple way.

In beginning of the first step of the calibration, a box called "Option" was created and presents the process (Figure 9). In Figure 9, there is a star in front of the line « $setglobal tariffadvalorem YES », meaning that the line is inactive. Therefore, in this case, specific tariffs are used. In order to use ad valorem tariffs, we need to delete this star and place it in front of the line « $setglobal tariffspecific YES ». By doing so, the line « $setglobal tariffspecific YES » becomes inactive, and the line « $setglobal tariffadvalorem YES » is activated. Ad valorem tariffs are used in this case. This process can be repeated to change from using one tariff equivalent to another. Once the tariff option is defined, the rest of the program automatically adjusts so there is no need to modify any other part of the model.

By activating both lines, we can take into account the simultaneous existence of both types of tariffs.

Figure 9 presents the modifications that have to be implemented in GAMS in order to switch from using ad valorem tariffs to using specific duty tariffs, or vice versa.

*Figure 9 Ad valorem tariffs versus specific duty tariffs – Calibration 1*

7.2 *Applying the model to another dataset*

The model can also be used with a different set of countries or regions. Data on bilateral trade flows, supply, consumption, transportation costs, producer and consumer prices, and tariffs is necessary.

The only modification that has to be implemented in the GAMS program is the loading of the new input Excel files. Once the input Excel files are in the correct directory, they can be uploaded on GAMS, using

the command presented in Figure 10. The same process can also be repeated to upload new transportation costs and tariffs matrices.

Figure 10 Example uploading a new trade matrix
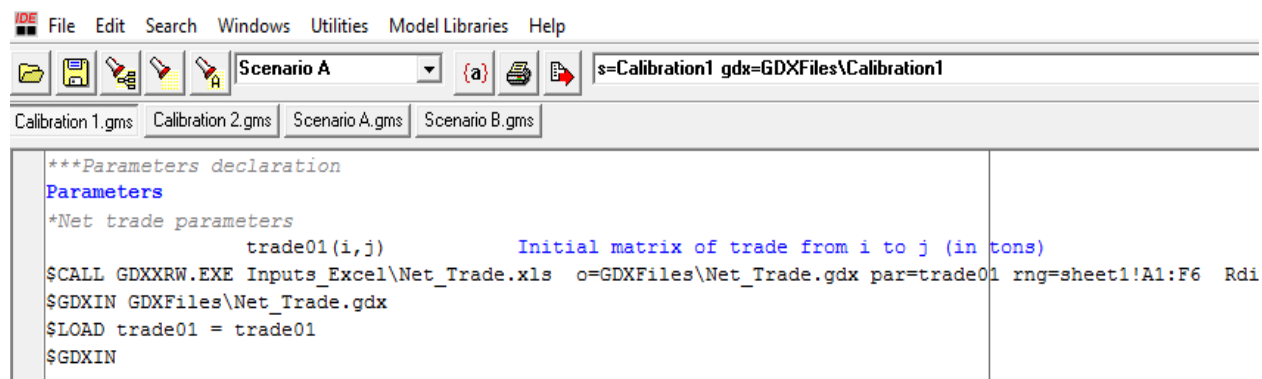
8. Limits of the spatial equilibrium model

As previously mentioned, the spatial equilibrium model is useful and is easy to use to study trade reallocation among countries after a shock on trade policies, or on transportation costs, or more broadly on a change in trade costs. More specifically, potential creation and diversion of trade flows between trading partners can be studied using a SEM.

However, three limits to the model can be identified. First, the model is a partial equilibrium model. Consequently, it does not take into account interdependence effects, as a general equilibrium model would. It is therefore limited to the study of a specific sector of the economy. Second, the estimation is based on very specific assumptions, as described in Section 2 of this document, especially on the hypothesis of homogeneous products. This hypothesis can be changed for the introduction of differentiated goods. Third, the spatial equilibrium model is generally used only with datasets that include border countries, or geographically near areas. It is complicated to justify the use of a spatial equilibrium model when trade partners are geographically far away from each other because the estimation of transportation costs becomes difficult. This last feature hence reduces the scope of application of a SEM.

However, this limit can be debated. Indeed, restricting the study to a set of geographically closed trading partners requires the use of disaggregated data, for example transportation costs between different cities of a country. The results obtained from a spatial equilibrium model are therefore more precise than those that could be obtained from a general equilibrium model. In addition, this specific feature of SEM increases transparency regarding the patterns of trade flows. Finally, extension possibilities are numerous with this type of model. For example, it can be employed to study the effects of quotas since those can be treated using a mixed complementarity problem.

It is important to keep in mind both the limits and advantages of such a model, in order to optimize its use.

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